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CONCERNING ALPHA AT SEVEN EARTH RADII

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SANTA MONICA • CALIFORNIA

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PREFACE

This brief study by a consultant to The RAND Corporation deals with the mathematical description of the motion of charged particles in the geomagnetic field. The work was performed for the National Aeronautics and Space Administration under Contract NASr-21(05).

ABSTRACT

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In a previous work, there was exposed an integral of the Størmer type which, although approximate, competently describes the motion of charged particles in the earth's magnetic field as described by Finch and Leaton. The present work shows, by a numerical example, that, at least for trapped radiation, the addition of a solar-wind cavity field introduces no complications that are quantitatively important for L values less than about 7 earth radii. The nature of the small numerical effect that is present at L = 7 makes it virtually certain that at significantly higher latitudes the approximate integral as known at present will fail significantly. A program is underway to calculate a correction term for the integral in case the particle in question is trapped.

Author

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INTRODUCTION

We have previously (Ray, 1963) derived and, in a preliminary way, studied an approximate Størmer integral that holds with good accuracy in much, if not all, of the phase space of a particle moving in a Finch and Leaton (1957) magnetic field. The present paper is a report of a numerical example involving an application of this integral to particles trapped in a magnetic-field model of the solar-wind cavity as obtained by Mead (1964). The example suggests the following two conclusions. In the first place, the deformation of the geomagnetic field, when the subsolar point of the cavity boundary is no nearer the earth's center than ten earth radii, is quantitatively unimportant in describing the behavior of trapped particles with $L \leq 7$ earth radii. For trapped particles with L values significantly larger than 7, it seems obvious that the deformation will become important. For $L = 7$, the numerical example shows that on the earth, a curve of constant-dipole magnetic latitude is a closer approximation to a curve of constant integral invariant than is a curve of constant α (Ray, 1963). Since the integral of Størmer type in question reduces to the constancy of α for a trapped particle, and since a curve of constant-dipole latitude obviously works poorly as the locus of the mirror point of a trapped particle at really high latitudes, it seems evident that for high-latitude trapped particles the approximate integral as now known will furnish only poor accuracy.

We therefore advocate the use of the integral in question for $L \leq 7$ and are pursuing a correction for use at higher latitudes, where the solar-wind cavity becomes important.

THE NUMERICAL EXAMPLE

We adopt as a model of the solar wind cavity the simplified Mead (1964) magnetic field, namely

$$B_r = -0.62\, r^{-3} \cos\theta - \bar{g}_1^0 \cos\theta - 2\sqrt{3}\, \bar{g}_2^1\, r \sin\theta \cos\theta \cos\varphi \quad ,$$

$$B_\theta = -0.31\, r^{-3} \sin\theta + \bar{g}_1^0 \sin\theta - \sqrt{3}\, \bar{g}_2^1\, r\, (2\cos^2\theta - 1) \cos\varphi \quad ,$$

$$B_\varphi = \sqrt{3}\, \bar{g}_2^1\, r \cos\theta \sin\varphi \quad ,$$

where r, θ, φ, are the usual spherical coordinates with r in earth radii,

$$\bar{g}_1^0 = -0.2515\, r_b^{-3} \quad ,$$

$$\bar{g}_2^1 = 0.1215\, r_b^{-4} \quad ,$$

and r_b is the distance, in earth radii, along the earth--sun line, from the earth's center to the cavity boundary. According to Mead, this model fits his calculations well when $r \le 0.7\, r_b$. We adopt $r_b = 10$.

We test the accuracy of the first integral in question by measuring the departure of the trajectory it predicts from that predicted by the usual adiabatic invariants. That is, we compare a curve of constant α with a curve of constant I, where I is the adiabatic integral invariant. The quantity I is not an exact integral, any more than α is. It is so, however, that the failure of I to be an integral can be reduced as much as desired by choosing to apply it to particles of sufficiently low rigidity, while the failure of α depends on properties of the magnetic field. The comparison is therefore appropriate.

We suppose that there is a trapped particle that initially spirals about that line of force that intersects the earth's surface at $\theta = 22^{\circ}.207$, $\varphi = 90^{\circ}$ and that has its mirror point at the earth's surface. The first two columns of the table give the curve on the earth's surface corresponding to I = 15.3.

Mirror Point Locus (I = 15.3)

φ ($^{\circ}$)	0	30	60	90	120	150	180
θ ($^{\circ}$)	22.52	22.48	22.36	22.21	22.06	21.95	21.91

This curve passes through the point in question, and is the path of the mirror point of our trapped particle. Strictly, the curve should be that on a surface of constant magnetic-field strength. The difference is insignificant for this field model. If the field were that of a pure dipole, so that $\bar{g}_1^0 = \bar{g}_2^1 = 0$, the locus of the mirror point would be the curve $\theta = 22^{\circ}.207$. It is seen that the departure from this behavior is nowhere appreciable. If the integral being investigated were exact, then according to arguments given previously (Ray, 1963), the equatorial crossings of the trapped particle would, in the equatorial plane, trace out a curve of constant B. This equatorial curve can then be projected along the lines of force that pass through it so as to produce a curve on the earth's surface. An inspection of the field model shows that this curve must be of the same shape as the curve in the table, having its maximum excursion from $\theta = 22^{\circ}.207$ at $\varphi = 0$ and at $\varphi = 180^{\circ}$, with the excursions about equal (since the deformation is not a large fraction of dipole field) and of opposite signs. Consequently, the following calculation was carried out on a computer. A line of force was computed from the point $r = 1$, $\theta = 22\overset{\circ}{.}207$, $\varphi = 90^{\circ}$ to the equatorial plane. The value of B there was calculated. It was then found that this same value of B was obtained by calculating a line of force from the point $r = 1$, $\theta = 23\overset{\circ}{.}176$, $\varphi = 0$ to the equatorial plane and calculating B at that equatorial point. This value, $23\overset{\circ}{.}176$ is about twice as far from the value $22\overset{\circ}{.}517$ on the constant I curve as is the value $22\overset{\circ}{.}207$ on the constant θ curve. The discrepancy is not large enough to be important for most purposes, but for larger L values, it seems likely that it will become steadily more serious. It is clear that for $L \leq 7$, however, the present theory is adequate. It is also trivial in the present case, where the neglect of the deformation results in the use of a pure dipole field. We draw the conclusion, however, that one is justified in neglecting the cavity deformation when $L \leq 7$; also when the higher-multipole terms of the earth's internal field are taken into account.

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